Chapter 9 Study Guide (PreCalc) Name: Key Hour: ___

Section 9.1: Polar Coordinates

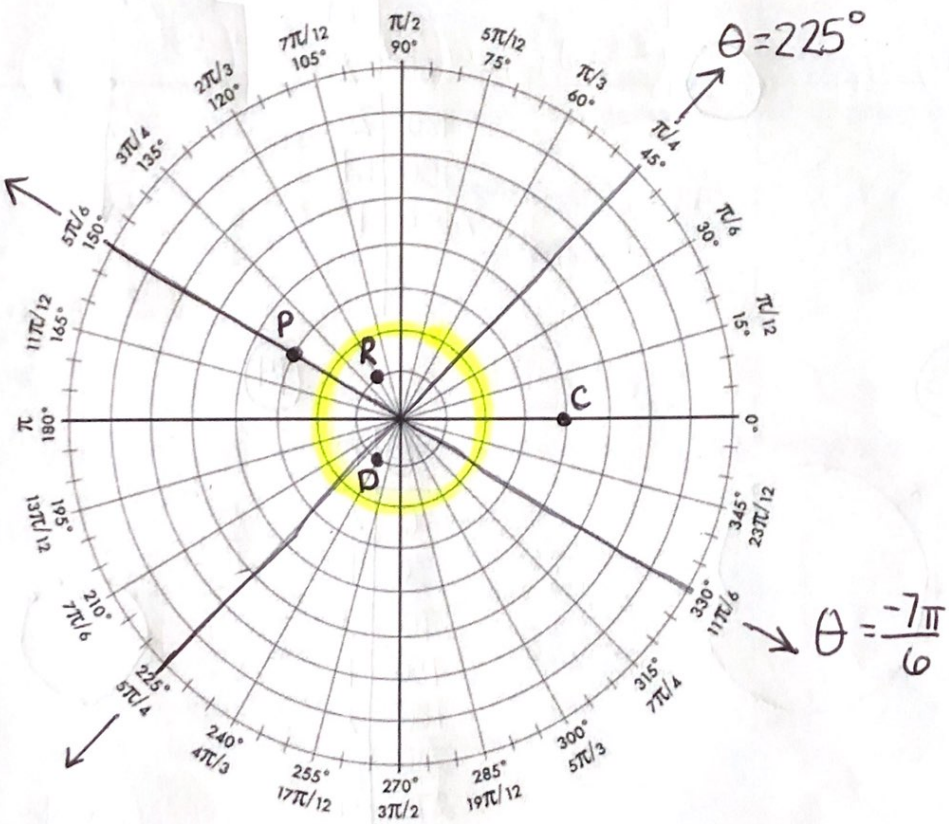
1) How can you find additional polar coordinates that have the same position as the original coordinate?

- Add/Subtract $2\pi/360°$
- Add/subtract $\pi/180°$ and make r the opposite sign

Graph each point/equation on the same polar grid below.

2) R(1, 120°) 3) P(3, -210°) 4) C(-4, π) 5) D(-1, $-\frac{5\pi}{3}$)

6) r = 2 7) $\theta = 225°$ 8) $\theta = \frac{-7\pi}{6}$



Find the distance between each pair of points. **You WILL NOT have this formula on the test!**

9) (4, -315°) and (1, 60°)

1 2

$\sqrt{1^2+4^2-2(1)(4)\cos(60°+315°)}$

$\sqrt{17-8\cos(375°)}$

$\sqrt{9.272} = \boxed{3.04}$

10) (7, $\frac{-\pi}{3}$) and (1, $\frac{2\pi}{3}$)

1 2

$\sqrt{1^2+7^2-2(1)(7)\cos\left(\frac{2\pi}{3}+\frac{\pi}{3}\right)}$

$\sqrt{50-14\cos(\pi)}$

$\sqrt{50+14} = \boxed{8}$

$\sqrt{(r_1)^2+(r_2)^2-2r_1r_2\cos(\theta_2-\theta_1)}$

9.2: Graphs of Polar Equations

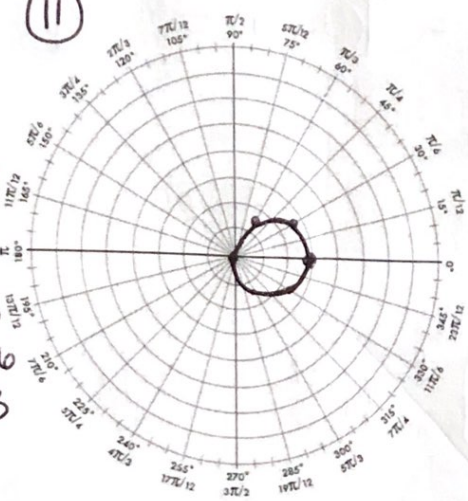
Graph the following on the polar grids provided. Remember, just like on your quiz, you'll be asked to provide a table with the points you used to graph your equation. Just like with the quiz, you will be given a chart with the different polar curves.

$r = -1/\cos\theta$

11) $r = 3\cos\theta$ 12) $r = -\sec\theta$ 13) $r = 4 - 3\cos\theta$ 14) $r = 2 - 2\sin\theta$

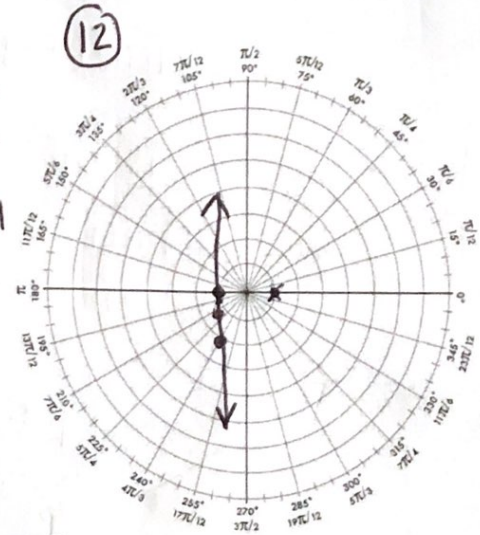
11

θ	r
0	3
30	2.6
60	1.5
90	0
120	-1.5
150	-2.6
180	-3



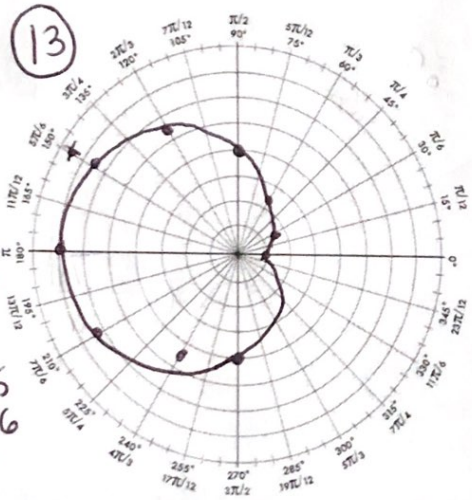
12

θ	r
0	-1
30	-1.1
60	-2
120	2
150	1.1
180	1



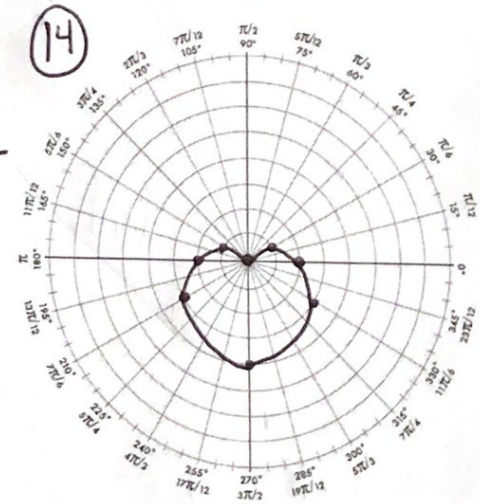
13

θ	r
0	1
30	1.4
60	2.5
90	4
120	5.5
150	6.6
180	7



14

θ	r
0	2
30°	1
90°	0
150°	1
180°	2
210°	3
270°	4
330°	3
360°	2



9.3: Polar and Rectangular Forms of Equations

15) How do you convert Polar to Rectangular (x,y) coordinates?

$x = r\cos\theta$

$y = r\sin\theta$

16) How do you convert Rectangular (x,y) to Polar Coordinates? What do you need to keep in mind when finding θ? (aka: when do you need to add pi/180°?)

$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ (if $x > 0$) $\theta = \tan^{-1}\left(\frac{y}{x}\right) + 180°/\pi$ (if $x < 0$)

Find the rectangular coordinates for each point with given polar coordinates. Found to the nearest hundredth, if possible.

17) (2, 45°)

$x = 2\cos 45$

$y = 2\sin 45$

$\boxed{(\sqrt{2}, \sqrt{2})}$

18) (-2, 270°)

$x = -2\cos 270°$

$y = -2\sin 270°$

$\boxed{(0, 2)}$

19) $(-1, \frac{\pi}{6})$

$x = -1\cos \pi/6$

$y = -1\sin \pi/6$

$\boxed{(-\sqrt{3}/2, -1/2)}$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \le \theta \le 2\pi$. Found to the nearest hundredth. Theta should be in **radians**.

20) (-1, 5)

$r = \sqrt{1+25}$ $\quad \theta = \tan^{-1}\left(\frac{5}{-1}\right) + 180°$

$r = \sqrt{26} = 5.1$ $\quad \theta = 101.3°$

$\left(5.1, \frac{101\pi}{180}\right)$ or $\left(-5.1, \frac{282\pi}{180}\right)$

21) (3, 7)

$\theta = \tan^{-1}\left(\frac{7}{3}\right)$

$r = \sqrt{9+49}$

$r = \sqrt{58}$ or 7.6 $\quad \theta = 66°$ or $\frac{66\pi}{180}$

$\left(7.6, \frac{66\pi}{180}\right)$ or $\left(-7.6, \frac{246\pi}{180}\right)$

22) (-2, 2)

$r = \sqrt{4+4}$

$r = \sqrt{8}$ or 2.8

$\theta = \tan^{-1}\left(\frac{2}{-2}\right) + \pi$

$\theta = -\pi/4 + \pi$

$\theta = 3\pi/4$

$(2.8, 3\pi/4)$ or $(-2.8, \frac{7\pi}{4})$

Write each equation in rectangular form, and identify its graph. **On the test, you may be asked to graph them as well!**

23) r = 5 CIRCLE

$\sqrt{x^2+y^2} = 5$

$\boxed{x^2+y^2 = 25}$

24) r = -4sin θ CIRCLE

$r^2 = -4r\sin\theta$

$x^2+y^2 = -4y$

$\boxed{x^2+y^2+4y = 0}$

Write each equation in polar form, and identify its graph. **On the test, you may be asked to graph them as well!**

25) y = 10 LINE

$r\sin\theta = 10$

$\boxed{r = \frac{10}{\sin\theta} \text{ or } r = 10\csc\theta}$

26) $(x + 5)^2 + y^2 = 25$ CIRCLE

$(r\cos\theta + 5)^2 + r^2\sin^2\theta = 25$

$(r^2\cos^2\theta + 10r\cos\theta + 25) + r^2\sin^2\theta = 25$

$r^2\cos^2\theta + r^2\sin^2\theta = -10r\cos\theta$

$r^2(\cos^2\theta + \sin^2\theta) = -10r\cos\theta$

$r^2 = -10r\cos\theta$

$\boxed{r = -10\cos\theta}$

9.5: Complex Numbers and DeMoivre's Theorem

27) How do you find the absolute value of a complex number?

Given $a+bi$, $\sqrt{a^2+b^2}$

28) How do you convert a complex number to polar form?

$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$ or $\theta = \tan^{-1}\left(\frac{b}{a}\right) + \pi$

$x > 0$ $\quad x < 0$

29) How do you convert a complex number in polar form back to rectangular form? (a + bi)?

Given $r[\cos\theta + i\sin\theta]$

→ Find $\cos\theta$ and $\sin\theta$

→ Distribute r

30) What are the product and quotient formula for Complex Numbers in Polar Form? **You will need to have these memorized for the test!**

Product: $r_1 r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

Quotient: $\frac{r}{r_2}[\cos(\theta_2-\theta_1) + i\sin(\theta_2-\theta_1)]$

31) What is DeMoivre's Theorem? **You will need to have these memorized for the test as well!**

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

32) Express -2 + 4i in polar form. $\rightarrow$ $4.47[\cos 116.57° + i\sin 116.57°]$

$r = \sqrt{(-2)^2+(4)^2}$ $\quad r = \sqrt{20}$ or 4.47 $\quad \theta = \tan^{-1}\left(\frac{4}{-2}\right) + 180°$

$r = \sqrt{4+16}$ $\quad \theta = 116.57°$

33) Express $3(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$ in rectangular form.

$3\left(-\frac{1}{2} + \frac{-\sqrt{3}}{2}i\right) = \boxed{-\frac{3}{2} + \frac{-3\sqrt{3}}{2}i}$

34) Find the product or quotient, and express it in rectangular form.

a) 5(cos135° + isin135°) • 2(cos45° + isin45°)

$10[\cos 180 + i\sin 180]$

$-10 + 0i$

$\boxed{-10}$

b) 1/2(cos60° + isin60°) ÷ 3(cos30° + isin30°)

$\frac{0.5}{3}[\cos 30° + i\sin 30°]$

$\frac{1}{6}[0.866 + 0.5i]$

$\boxed{0.144 + 0.083i}$

35) Find each power, and express it in rectangular form.

a) $(3 - 5i)^4$

$r = \sqrt{9+25}$ $\quad \theta = \tan^{-1}\left(\frac{-5}{3}\right)$

$r = \sqrt{34}$ or 5.83 $\quad \theta = -59.03°$

$[5.83(\cos -59.03 + i\sin -59.03]^4$

$5.83^4(\cos[4 \cdot -59.03] + i\sin[4 \cdot -59.03])$

$1155.2(\cos -236.12 + i\sin -236.12)$

$1155.2(-0.55 + 0.83i)$

$\boxed{-635.36 + 958.81i}$

b) $[2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})]^3$

$2^3(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$

$8\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$\boxed{-4\sqrt{2} + 4\sqrt{2}i}$