Chapter 6 Study Guide - Due Monday, March 18th

6.2: Matrix Multiplication, Inverses, Determinants
- Make sure you review…
  - How to multiply matrices on your calculator
  - When you can multiply matrices
  - How to find the inverse of a matrix
  - How to find the determinant of a matrix

**Inverses are only possible with __Square matrices (ex: 2×2, 3×3)__

** If your determinant equals zero, then __no inverse exists__

** If your determinant equals anything other than zero, then __there is an inverse__

Find AB and BA, if possible.

1)
$A = \begin{bmatrix} 8 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 19 & -54 \end{bmatrix}$

$BA = \emptyset$

2) 3×1
$A = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ 2×3 $B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

$AB = \emptyset$

$BA = \begin{bmatrix} 6 \\ 85 \end{bmatrix}$

3)
$A = \begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$

$AB = \begin{bmatrix} -9 & 6 & 12 \\ -41 & -14 & 65 \end{bmatrix}$ $BA = \emptyset$

Determine whether A and B are inverse matrices. If they aren't, find the inverse of A.

4)
$A = \begin{bmatrix} 9 & 2 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 \\ 5 & -9 \end{bmatrix}$

$\boxed{yes}$

5)
$A = \begin{bmatrix} 9 & -7 \\ 8 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -6 \\ 4 & 10 \end{bmatrix}$

$\boxed{No}$ — $A^{-1} = \begin{bmatrix} -0.5 & 0.6 \\ -0.7 & 0.8 \end{bmatrix}$

Find A^{-1}, if it exists.

6)
$A = \begin{bmatrix} 5 & 2 & -1 \\ 4 & 7 & -3 \\ 1 & -5 & 2 \end{bmatrix}$ $\emptyset$

7)
$A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix}$ $\begin{bmatrix} -34 & 29 & 9 \\ 7 & -6 & -2 \\ -12 & 10 & 3 \end{bmatrix}$

Find the determinant of each matrix. Then, if it has an inverse, write the inverse. Round answers to the nearest tenth!

8)
$\begin{bmatrix} 12 & -9 \\ -4 & 3 \end{bmatrix}$ $\emptyset$

$det = 0$

9)
$\begin{bmatrix} 3 & 1 & -2 \\ 8 & -5 & 2 \\ -4 & 3 & -1 \end{bmatrix}$

$det = -11$

$\begin{bmatrix} 0.1 & 0.5 & 0.7 \\ 0 & 1 & 2 \\ -0.4 & 1.2 & 2.1 \end{bmatrix}$

6.3: Solving Linear Systems using Inverses
- Make sure you review…
  - How to solve systems of equations using inverse matrices
  - Solving Linear-Nonlinear Systems by Graphing
  - Solving Linear-Nonlinear Systems by Substitution
  -

Write the following system of equations in matrix form, AX = B. Then, solve each system of equations.

10) $5x - 2y = 11$ $\boxed{x = 3}$
$-4x + 7y = 2$ $\boxed{y = 2}$

$$\begin{bmatrix} 5 & -2 \\ -4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

11) $-3x + 5y = 33$ $\boxed{x = -1}$
$2x - 4y = -26$ $\boxed{y = 6}$

$$\begin{bmatrix} -3 & 5 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -26 \end{bmatrix}$$

12) $-4x + y = 19$ $\boxed{x = -4}$
$3x - 2y = -18$ $\boxed{y = 3}$

$$\begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -18 \end{bmatrix}$$

13) $2x + y - z = -13$ $\boxed{x = -6}$
$3x + 2y - 4z = -36$ $\boxed{y = 7}$
$x + 6y - 3z = 12$ $\boxed{z = 8}$

$$\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -4 \\ 1 & 6 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -36 \\ 12 \end{bmatrix}$$

14) $x + 2y = 12$ $\boxed{x = 6}$
$3y - 4z = 25$ $\boxed{y = 3}$
$x + 6y + z = 20$ $\boxed{z = -4}$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -4 \\ 1 & 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 25 \\ 20 \end{bmatrix}$$

15) $9x + 7y = -30$ $\boxed{x = -1}$
$8y + 5z = 11$ $\boxed{y = -3}$
$-3x + 10z = 73$ $\boxed{z = 7}$

$$\begin{bmatrix} 9 & 7 & 0 \\ 0 & 8 & 5 \\ -3 & 0 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30 \\ 11 \\ 73 \end{bmatrix}$$

6.4: Partial Fractions
- Make sure you review…
  - Finding Partial Fraction Decomposition when the degree in the numerator is SMALLER than the degree in the denominator
  - Find Partial Fraction Decomposition when the degree in the numerator is GREATER THAN or EQUAL TO the degree in the denominator

Find the partial fraction decomposition of each rational expression.

16) $\dfrac{x+1}{x^2 + 5x + 6}$

$$\dfrac{-1}{x+2} + \dfrac{2}{x+3}$$

17) $\dfrac{x+13}{x^2 + 7x + 12}$

$$\dfrac{+10}{x+3} + \dfrac{-9}{x+4}$$

Find the partial fraction decomposition of each improper rational expression.

18) $\dfrac{3x^2 + 12x + 4}{x^2 + 2x}$

$$3 + \dfrac{2}{x} + \dfrac{-4}{x+2}$$

19) $\dfrac{x^4 - 3x^3 + x^2 - 9x + 4}{x^2 - 4x}$

$$\left(x^2 + x + 5\right) + \dfrac{12}{x} + \dfrac{-1}{x-4}$$
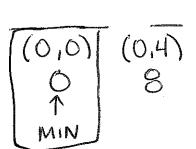
6.5: Linear Optimization
- Make sure you review…
    - How to graph the constraints
    - How to determine the
      maximum/minimum values

Find the maximum and minimum values of the
objective function f(x, y) and for what values of x
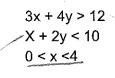and y they occur, subject to given constraints.

20) $f(x, y) = 8x + 2y$

$4x + 5y < 35$ → $5y \le -4x + 35$
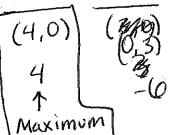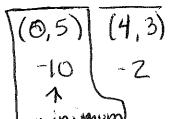$x + 5y < 20$ $y \le -\frac{4}{5}x + 7$
 $y > 0$
 $x > 0$ $5y < -x + 20$
                    $y < \frac{-1}{5}x + 4$

$0 < 0 + 4$
$0 < 4$ ☉

(0,0)	(0,4)	(9,0)	(5,3)
0	8	72	46
↑		↑	
MIN		Max	

21) $f(x, y) = x - 2y$

$3x + 4y > 12$
$x + 2y < 10$
$0 < x < 4$

$4y > -3x + 12$
$y > -\frac{3}{4}x + 3$
$0 > 3$

$2y < -x + 10$
$y < -\frac{1}{2}x + 5$
$0 < 5$

(4,0)	(3,0) (0,3)	(0,5)	(4,3)
4	-6	-10	-2
↑		↑	
Maximum		minimum	