

# FALL SEMESTER FINAL EXAM STUDY GUIDE - PART TWO

## Precalculus

Name: Key

\*\*Note: Questions on this study guide are not exact replicas of what will be on the final - the concepts covered on this study guide will be on the final, however. It's recommended to look at/review old tests/quizzes as well\*\*

### 2.1 - Power and Radical Functions

Analyze the following functions. Determine the Domain and Range, intercepts, end behavior, continuity,

1)  $f(x) = 5x^2$

2)  $-0.5x^7 = g(x)$

Domain:  $(-\infty, \infty)$

Domain:  $(-\infty, \infty)$

Range:  $(0, \infty)$

Range:  $(-\infty, \infty)$

x-int:  $(0, 0)$

x-int:  $(0, 0)$

y-int:  $(0, 0)$

y-int:  $(0, 0)$

EB:  $\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = \infty$

EB:  $\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = -\infty$

Continuity: Continuous on  $(-\infty, \infty)$

Continuity: Continuous on  $(-\infty, \infty)$

3)  $f(x) = -8x^{1/3}$

4)  $g(x) = 10x^{1/2}$

Domain:  $(-\infty, \infty)$

Domain:  $(0, \infty)$

Range:  $(-\infty, \infty)$

Range:  $(0, \infty)$

x-int:  $(0, 0)$

x-int:  $(0, 0)$

y-int:  $(0, 0)$

y-int:  $(0, 0)$

EB:  $\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = -\infty$

EB:  $\lim_{x \rightarrow \infty} f(x) = \infty$

Continuity:

Continuity: Continuous on

continuous on  
 $(-\infty, \infty)$

$(\infty, 0)$

Solve each equation.

5)  $\sqrt[4]{(4x + 164)^3} + 36 = 100$

6)  $7 + \sqrt{(-36 - 5x)^5} = 250$

$$\sqrt[4]{(4x + 164)^3} = 64$$

$$\sqrt{(-36 - 5x)^5} = 243$$

$$(4x + 164)^3 = 64^4$$

$$(-36 - 5x)^5 = 243^2$$

$$\sqrt[3]{(4x + 164)^3} = \sqrt[3]{64^4}$$

$$\sqrt[5]{(-36 - 5x)^5} = \sqrt[5]{243^2}$$

$$4x + 164 = 256$$

$$-36 - 5x = 9$$

$$4x = 92$$

$$-5x = 45$$

$$x = -9$$

## 2.2 - Polynomial Functions

- 7) What does the graph of a polynomial function look like? Defined, Continuous for all real numbers, smooth, rounded turns
- 8) What does the graph of a non-polynomial function look like? Contains breaks, holes, gaps, and/or sharp corners
- 9) What two parts of the polynomial does the leading term test use? Leading Coefficient + Degree

10) A polynomial with degree  $n$  has  $n$  real zeros and  $n-1$  turning points.

11) Describe the end behavior of each polynomial function, and explain your answer using the leading term test.

a)  $f(x) = -5x^7 + 6x^4 + 8$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = -\infty$$

leading coefficient: negative  
degree: odd

b)  $h(x) = 8x^2 + 5 - 4x^3$

$$\lim_{x \rightarrow -\infty} h(x) = \infty \quad \lim_{x \rightarrow \infty} h(x) = \infty$$

leading coefficient: positive  
degree: even

12) State the number of possible real zeros and turning points for each function. Then, determine all of the real zeros by factoring.

a)  $f(x) = x^5 + 3x^4 + 2x^3$

real zeros: 5

turning points: 4

$$x^3(x^2 + 3x + 2)$$

$$x^3(x+2)(x+1)$$

Zeros:  
-2, -1, 0  
mult. 3

b)  $f(x) = 4x^8 + 16x^4 + 12$

real zeros: 8

turning points: 7

$$4(x^4)^2 + 16(x^4) + 12$$

$$4m^2 + 16m + 12$$

$$(4m+12)(m+1)$$

No real zeros!

Let

$$m = x^4$$

$$m = -1 \quad x^4 = -1 \quad \text{X}$$

$$m = -3 \quad x^4 = -3 \quad \text{X}$$

13) For each function, apply the leading term test, and determine the zeros and state the multiplicity of any repeated zeros.

a)  $f(x) = 3x^3 - 3x^2 - 36x$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

leading coefficient: positive  
degree: odd

$$x(3x^2 - 3x - 36)$$

$$3x(x^2 - x - 12)$$

$$3x(x-4)(x+3)$$

$$x=0 \quad x=4 \quad x=-3$$

b)  $f(x) = x^2(x-4)(x+2)$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

leading coefficient: positive  
degree: even

$$x=0 \quad x=4 \quad x=-2$$

(mult: 2)

## 2-3 The Remainder and Factor Theorems

- 14) What is the remainder theorem? If a polynomial  $f(x)$  is divided by  $x - c$ , the remainder is  $f(c)$
- 15) What is the factor theorem? A polynomial  $f(x)$  has a factor  $(x - c)$  if and only if  $f(c) = 0$

16) Divide the following using synthetic division or long division

a)  $(2x^4 - 5x^2 + x - 2) / (x + 2)$

$$\begin{array}{r} \boxed{-2} \mid 2 & 0 & -5 & 1 & -2 \\ & \downarrow & -4 & 8 & -6 & 10 \\ \hline & 2 & -4 & 3 & -5 & 8 \end{array}$$

$$2x^3 - 4x^2 + 3x - 5 \quad R: 8$$

c)  $(x^4 - x^3 + 3x^2 - 6x - 6) / (x - 2)$

$$\begin{array}{r} \boxed{2} \mid 1 & -1 & 3 & -6 & -6 \\ & \downarrow & 2 & 2 & 10 & 8 \\ \hline & 1 & 1 & 5 & 4 & \boxed{2} \end{array}$$

$$x^3 + x^2 + 5x + 4 \quad R: 2$$

b)  $(4x^3 + 3x^2 - x + 8) / (x - 3)$

$$\begin{array}{r} \boxed{3} \mid 4 & 3 & -1 & 8 \\ & \downarrow & 12 & 15 & 132 \\ \hline & 4 & 15 & 44 & \boxed{140} \end{array}$$

$$4x^2 + 15x + 44$$

$$R: 140$$

d)  $(3x^4 - 9x^3 - 24x - 48) / (x - 4)$

$$\begin{array}{r} \boxed{4} \mid 3 & -9 & 0 & -24 & -48 \\ & \downarrow & 12 & 12 & 48 & 96 \\ \hline & 3 & 3 & 12 & 24 & \boxed{48} \end{array}$$

$$3x^3 + 3x^2 + 12x + 24 \quad R: 48$$

17) Use the factor theorem to determine if the binomials given are factors of  $f(x)$ . Use the binomials that are factors to write a factored form of  $f(x)$ .

a)  $f(x) = 2x^3 - x^2 - 41x - 20$ ;  $(x + 4)$ ,  $(x - 5)$

$$\begin{array}{r} \boxed{-4} \mid 2 & -1 & -41 & -20 \\ & \downarrow & -8 & 36 & 20 \\ \hline & 2 & -9 & -5 & \boxed{0} \end{array}$$

$$\begin{array}{r} \boxed{5} \mid 2 & -1 & -41 & -20 \\ & \downarrow & 10 & 45 & 20 \\ \hline & 2 & 9 & 4 & \boxed{0} \end{array}$$

b)  $f(x) = x^4 + 2x^3 - 5x^2 + 8x + 12$ ;  $(x - 1)$ ,  $(x + 3)$

$$\begin{array}{r} \boxed{1} \mid 1 & 2 & -5 & 8 & 12 \\ & \downarrow & 1 & 3 & -2 & 6 \\ \hline & 1 & 3 & -2 & 6 & \boxed{18} \end{array}$$

$$\begin{array}{r} \boxed{-3} \mid 1 & 2 & -5 & 8 & 12 \\ & \downarrow & -3 & 3 & 6 & -42 \\ \hline & 1 & -1 & -2 & 14 & \boxed{-30} \end{array}$$

## 2-4 Zeros of Polynomial Functions

17) What is the Rational Zero Theorem?

It allows you to see all the possible rational zeros, of a polynomial by using the factors of the constant and the factors of the leading coefficient.

18) List all possible rational zeros of each function by using the rational zero theorem.

a)  $g(x) = x^4 + 4x^3 - 12x - 9$

$$\begin{array}{l} \text{Factors of } 9 = \pm 1, \pm 3, \pm 9 \\ \text{Factors of } 1 = \pm 1 \end{array}$$

$$\boxed{\pm 1, \pm 3, \pm 9}$$

b)  $g(x) = 3x^4 - 18x^3 + 2x - 21$

$$\begin{array}{l} \text{Factors of } 21 = \pm 1, \pm 3, \pm 7, \pm 21 \\ \text{Factors of } 3 = \pm 1, \pm 3 \end{array}$$

$$\boxed{\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{3}, \pm \frac{7}{3}}$$

## 2-5 Rational Functions

19) How do you know if your function has an asymptote?

Gaps in the domain; if it approaches an H/V line  
 Set the denominator of a rational zero equal to zero and solve.

20) How do you find a vertical asymptote?

If the degree on top is greater than the degree on bottom.

21) How do you know if your horizontal asymptote is  $y = 0$ ?

If the degree on top is less than the degree on bottom.

22) How do you know if you don't have a horizontal asymptote?

If the degree on top is equal to the degree on bottom.

23) If the degree on top and on bottom are equal, what is your horizontal asymptote?

$$y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$$

24) Find the domain and range of each function, the intercepts, and the equations of the vertical or horizontal asymptotes, if any.

a)  $f(x) = \frac{x^3 - 8}{x + 4}$  y-int: (0, -2)

D:  $(-\infty, -4) \cup (-4, \infty)$

R:  $(-\infty, \infty)$  x-int: (2, 0)

b)  $f(x) = \frac{x - 8}{x^2 + 4x + 5}$

D:  $(-\infty, \infty)$

R:  $(-\infty, 0)$

y-int: (0, -1.6)

x-int: (8, 0)

asy:  $x = 2, x = -2, y = 1$

asy:  $y = 0$

c)  $f(x) = \frac{x^2 - 4x - 21}{x^3 + 2x^2 - 5x - 6}$

d)  $f(x) = \frac{x^3}{x+3}$

D:  $(-\infty, -3) \cup (-3, -1) \cup (-1, 2) \cup (2, \infty)$

D:  $(-\infty, -3) \cup (-3, \infty)$

R:  $(-\infty, 1) \cup (2.9, \infty)$

R:  $(-\infty, \infty)$

y-int: (0, 3.5) x-int: 7, 0

x-int: (0, 0) y-int: (0, 0)

asy:  $x = -1, x = 2, y = 0$

asy:  $x = -3$

25) Solve each equation.

a)  $y + \frac{6}{y} = 5$

$y(y + \frac{6}{y}) = 5y$

$y^2 + 6 = 5y$

$y^2 - 5y + 6 = 0$

$(y-2)(y-3) = 0$

$y = 2$
$y = 3$

b)  $\frac{4}{x-2} - \frac{2}{x} = \frac{14}{x^2 - 2x}$

$(\frac{x}{x} \cdot \frac{4}{x-2}) - (\frac{x-2}{x-2} \cdot \frac{2}{x}) = \frac{14}{x^2 - 2x}$

$\frac{4x}{x^2 - 2x} - \frac{2x - 4}{x^2 - 2x} = \frac{14}{x^2 - 2x}$

$4x - (2x - 4) = 14$

$2x + 4 = 14$

$2x = 10$

$x = 5$
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