FALL SEMESTER FINAL EXAM STUDY GUIDE - PART ONE

[PRECALCULUS] Name: Key

Note: Questions on this study guide are not exact replicas of what will be on the final - the concepts covered on this study guide will be on the final, however. It's recommended to look at/review your notes, old tests, and old quizzes as well

0.2: Imaginary Numbers

1) $i = \sqrt{-1}$

2) $i^2 = -1$

$i^{17} = i \cdot i^{16} = i \cdot (i^2)^8 = i$

$i^{14} = (i^2)^7 = -1$

$i^5 = i \cdot i^4 = i \cdot 1 = i$

Simplify.

3) $i^{52} = (i^2)^{26} = (-1)^{26}$

1

4) $i^{31} = i \cdot i^{30}$

$i \cdot (i^2)^{15}$

$i \cdot (-1)^{15} = -i$

5) $i^{14} + i^5 - i^{17}$

$-1 + 1i + -1i$

-1

6) (5 - 3i) + (-2 - 10i)

$\boxed{3 - 13i}$

7) (18 + 6i) - (13 - 2i)

$\boxed{5 + 8i}$

9) (2 - 10i)(5 + 5i)

$10 + 10i - 50i - 50i^2$

$10 + -40i + 50$

$\boxed{60 - 40i}$

10) (23 + 10i)(31 - 12i)

$713 - 276i + 310i - 120i^2$

$713 + 34i + 120$

$\boxed{833 + 34i}$

11) $\frac{5+i}{6+i} \cdot \frac{(6-i)}{(6-i)} = \frac{30 - 5i + 6i - i^2}{36 - i^2} = \boxed{\frac{31+i}{37}}$

12) $\frac{3+4i}{1+5i} \cdot \frac{(1-5i)}{(1-5i)} = \frac{3 - 15i + 4i - 20i^2}{1 - 25i^2} = \boxed{\frac{23-11i}{26}}$

0.3: Quadratic Functions/Equations

13) How do you find the y-intercept of a quadratic function? Plug 0 into x and solve

14) How do you find the axis of symmetry? $x = -b/2a$

15) How do you find the vertex? The x-coordinate is $-b/2a$. The y-coordinate is found by taking x and plugging it into the function.

16) What are the different methods you can use to solve a quadratic? Quadratic Formula, Factoring, Complete the Square

17) Find the axis of symmetry, y-intercept, and vertex for the function $f(x) = 2x^2 - 8x - 5$. Then, solve.

y-int: $\boxed{-5}$

AoS: $x = 8/4 = 2$

$\boxed{x = 2}$

vertex: $\boxed{(2, -13)}$

$2(2)^2 - 8(2) - 5$

$8 - 16 - 5 = -13$

0.5: Solving Systems of Equations

18) Solve each system of equation using any method you prefer.

a) $3y = 4x + 6$
$2y = x - 1$

$3y = 4x + 6$
$-8y = -4x + 4$
$-5y = 10$
$\boxed{y = -2 \quad x = -3}$

b) $5x - y = 16$
$2x + 3y = 3$

$15x - 3y = 48$
$2x + 3y = 3$
$17x = 51$
$x = 3$
$2x + 3y = 3$
$6 + 3y = 3$
$3x = -3$
$x = -1$
$\boxed{(-1, 3)}$

c) $5x - 6y = 10$
$-2x + 3y = -7$

$5x - 6y = 10$
$-4x + 6y = -14$
$x = -4$
$\boxed{x = -4 \quad y = -5}$
$5(-4) - 6y = 10$
$-20 - 6y = 10$
$-6y = 30$
$y = -5$

19) How do you know if your system of equations will give you an infinite amount of solutions?
If your variables cancel out and you are left with two numbers that equal each other, then you'll have infinite solutions.

20) How do you know if your system of equations will give you no solutions?
If your variables cancel out and you are left with two numbers that don't equal each other, then you'll have no solutions.

21) The senior classes at Northview High School and Creston High School planned separate trips to St. Louis, MO. The senior class at Northview High rented and filled 1 van and 6 buses with 372 students. Creston High rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

$v + 6b = 372$
$4v + 12b = 780$

$-2v + -12b = -744$
$4v + 12b = 780$
$2v = 36$

$\boxed{v = 18 \quad b = 59}$

0.4: nth Roots and Real Exponents *(Oops...this should have been before the last section, but I didn't want to re-number the problems…)*

Simplify

22) $-\sqrt{196}$

-14

23) $\sqrt[3]{\frac{216}{125}}$

$6/5$

24) $\sqrt[4]{-81}$

$\varnothing$

25) $\sqrt[3]{-27x^9}$

$-3x^3$

26) $\sqrt{20x^{16}y^{41}}$

$\sqrt{20} \cdot \sqrt{x^{16}} \cdot \sqrt{y^{41}}$
$2\sqrt{5} \quad x^8 \quad \sqrt{y}\sqrt{y^{40}}$
$2\sqrt{5} \cdot x^8 \cdot y^{20}\sqrt{y}$

$\boxed{2x^8y^{20}\sqrt{5y}}$

27) $\frac{x^{5/4} \cdot x^{3/4}}{x^{1/4}} = \frac{x^{8/4}}{x^{1/4}} = x^{7/4} = \boxed{\sqrt[4]{x^7}}$

0.6: Matrix Operations

Find each of the following for $A = \begin{bmatrix} 10 & -6 \\ 7 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 10 & 20 & 30 \end{bmatrix}$, $C = \begin{bmatrix} 2x & 3y \\ -y & 10x \end{bmatrix}$, $D = \begin{bmatrix} -15 & 7 \end{bmatrix}$

28) A + C

$\begin{bmatrix} 10+2x & -6+3y \\ 7-y & -1+10x \end{bmatrix}$

29) 4C - 2A

$\begin{bmatrix} 8x & 12y \\ -4y & 40x \end{bmatrix} - \begin{bmatrix} 20 & -12 \\ 14 & -2 \end{bmatrix} = \begin{bmatrix} 8x-20 & 12y+12 \\ -4y-14 & 40x+2 \end{bmatrix}$

30) B - D → $\varnothing$

31) In order to add/subtract matrices, what must be true? They must have the same dimensions.

1.1: Functions

32) How do you know if a relation is a function?

- Method 1: Using the vertical line test
- Method 2: No repeats in the domain

33) Describe the set of numbers using set-builder notation <u>and</u> interval notation (if possible)

a) {7, 6, 5, 4, ...}

SBN: $\{x \mid x \le 7,\ x \in \mathbb{Z}\}$

IN: $(-\infty, 7]$

b) x > 12

$\{x \mid x > 12,\ x \in \mathbb{R}\}$

$[12, \infty)$

c) x > 9 or x < -1

$\{x \mid x > 9,\ x < -1,\ x \in \mathbb{R}\}$

$(-\infty, -1) \cup (9, \infty)$

34) Given $g(x) = 2x^2 + 18x - 14$ and $h(x) = \frac{4x+11}{3x^2+5x+1}$, find each function value.

a) g(9)

$g(9) = 2(9)^2 + 18(9) - 14$

$= 310$

b) g(3x)

$g(3x) = 2(3x)^2 + 18(3x) - 14$

$= 2(9x^2) + 54x - 14$

$= \boxed{18x^2 + 54x - 14}$

c) ~~f(-2)~~ h(-2)

$h(-2) = \frac{4(-2)+11}{3(-2)^2+5(-2)+1}$

1

d) ~~f(4t)~~ h(4t)

$\frac{4(4t)+11}{3(4t)^2+5(4t)+1}$

$\boxed{\frac{16t+11}{48t^2+20t+1}}$

1.2 - 1.5: Analyzing Graphs of Functions and Relations

35) Given $f(x) = 4x^4 + 10x^2$, answer the following:

a) Determine the Domain and Range D: $(-\infty, \infty)$ R: $(0, \infty)$

b) Find f(-2) and f(4) $f(-2) = 4(-2)^4 + 10(-2)^2 = 104$ $f(4) = 4(4)^4 + 10(4)^2 = 1184$

c) Find any X and Y intercepts x-int: $(0,0)$ y-int: $(0,0)$

d) Determine any symmetry the graph contains, and if that symmetry classifies the graph/function as odd or even Symmetry on y-axis (even function)

e) Continuity, and if it is discontinuous, determine the location and what type of discontinuity it is Continuous throughout domain

f) End Behavior $\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = \infty$

g) Where the graph is increasing/decreasing/constant on the graph Decreasing: $(-\infty, 0)$ Increasing: $(0, \infty)$

h) Find any relative or absolute maximums or minimums Minimum: $(0,0)$

i) Determine the average rate of change on [1, 2]

$\frac{f(2) - f(1)}{2 - 1} = \frac{104 - 14}{1} = 90$

$f(2) = 104$

$f(1) = 14$

36) Given the graph of g(x), answer the following:

a) Determine the Domain and Range D: $(-\infty, \infty)$ R: $(-\infty, 3)$

b) Find g(-2) and g(4) $g(-2) = 3$ $g(4) = -3$

c) Find any X and Y intercepts x-int: $(-5,0), (1,0)$ y-int: $(0,1)$

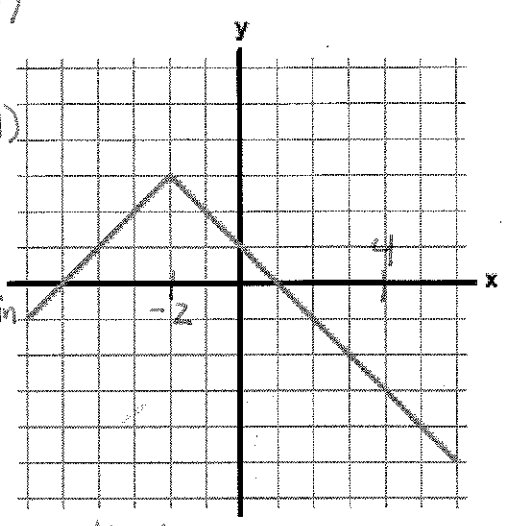
d) Determine any symmetry the graph contains, and if that symmetry classifies the graph/function as odd or even Line Symm. over $y = -2$

e) Continuity, and if it is discontinuous, determine the location and what type of discontinuity it is Continuous throughout domain

f) End Behavior $\lim_{x \to -\infty} g(x) = -\infty$ $\lim_{x \to \infty} g(x) = \infty$

g) Where the graph is increasing/decreasing/constant on the graph increasing: $(-\infty, -2)$ decreasing: $(-2, \infty)$

h) Find any relative or absolute maximums or minimums Maximum at $x = -2$



i) Determine the average rate of change on [1, 2]

$$\frac{f(2) - f(1)}{2 - 1} = \frac{-1 - 0}{2 - 1} = \frac{-1}{1} = -1$$

1.6: Function Operations and Composition of Functions

37) Given $f(x) = x^2 - 6x - 8$ and $g(x)$ $\sqrt{x}$, find (f + g)(x), (f - g)(x), (f*g)(x), and (f/g)(x).

$(f+g)(x) = x^2 - 6x - 8 + \sqrt{x}$ $(f \cdot g)(x) = (x^2 - 6x - 8)(\sqrt{x}) = x^2\sqrt{x} - 6x\sqrt{x} - 8\sqrt{x}$

$(f-g)(x) = x^2 - 6x - 8 - \sqrt{x}$ $(f/g)(x) = \frac{x^2 + 6x - 8}{\sqrt{x}} = \frac{x^2\sqrt{x} + 6x\sqrt{x} - 8\sqrt{x}}{x}$

38) Given f(x) = 3x + 1 and $g(x) = 10 - x^2$, find f(g(x)), g(f(x)), and f(g(3)).

$f(g(x)) = 3[10 - x^2] + 1 = 30 - 3x^2 + 1 = 31 - 3x^2$ $f(g(3)) = 31 - 3(3)^2 = 4$

$g(f(x)) = 10 - (3x+1)^2 = 10 - [9x^2 + 6x + 1] = -9x^2 - 6x + 9$

1.7: Inverse Relations and Functions

39) How do you know if a function has an inverse? A function has an inverse if it passes the horizontal line test. (HLT).

40) Determine if the following have inverse functions, and explain why/why not. If they do, state the inverse, and determine the inverse function's domain and range. Be sure to show all work.

a) f(x) = 10x - 12

$x = 10y - 12$
$x + 12 = 10y$

Yes; pass HLT

$f^{-1}(x) = \frac{x+12}{10}$

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

b) $g(x) = \sqrt{6 - x^2}$

No; doesn't pass HLT

c) $h(x) = \sqrt{x + 8}$

$x = \sqrt{y + 8}$

Yes; passes HLT

$x^2 - 8 = h^{-1}(x)$

D: $(-\infty, \infty)$ R: $(8, \infty)$

41) Show algebraically that f and g are inverse functions.

a) f(x) = -6x + 3

g(x) = (3 - x)/6

they are equal YES!

$x = \frac{3 - y}{6}$
$6x = 3 - y$

$6x - 3 = -y$
$-6x + 3 = y$

b) $f(x) = \frac{x+4}{x}$, $g(x) = \frac{4}{x-1}$

$x = \frac{4}{y-1}$
$(y-1)x = 4$
$y - 1 = 4/x$

$y = \frac{4}{x} + 1$
$y = \frac{4}{x} + \frac{x}{x}$
$= 4 + x/x$

they are equal YES!