

Algebra 2 Fall Semester Final Exam Study Guide - Chapter 3 and 4 (Part 2)

Name: KEY

3.1: Solving Systems of Equations

$$\begin{aligned} 1) \quad & -5x - y = 12 \\ + & \quad 7x + y = -16 \\ \hline & \quad 2x = -4 \\ & \quad x = -2 \end{aligned}$$

$$\begin{aligned} -14 + y = -16 \\ y = -2 \end{aligned}$$

$$\begin{aligned} 2) \quad & 3x - 2y = -4 \\ + & \quad -x + 2y = 4 \\ \hline & \quad 2x = -8 \\ & \quad x = -4 \end{aligned}$$

$$\begin{aligned} 3(-4) - 2y = -4 \\ -2y = 8 \\ y = -4 \end{aligned}$$

$$\begin{aligned} 3(-9x + 10y = -22) & \rightarrow -27x + 30y = -66 \\ (-7x - 8y = -18) 10 & \rightarrow -70x - 80y = -180 \\ -142x & = -356 \\ x & = 2.5 \\ -7(2.5) - 8y & = -18 \\ -8y & = -0.5 \\ y & = 0.0625 \end{aligned}$$

- 4) Andre and Paul each mow lawns. Andre charges a \$30 service fee and \$10 per hour. Paul charges a \$10 service fee and \$15 per hour. After how many hours will Andre charge the same amount? Write a system of equations to represent this situation.

$x = \# \text{ of hours}$

$y = \text{total amount charged}$

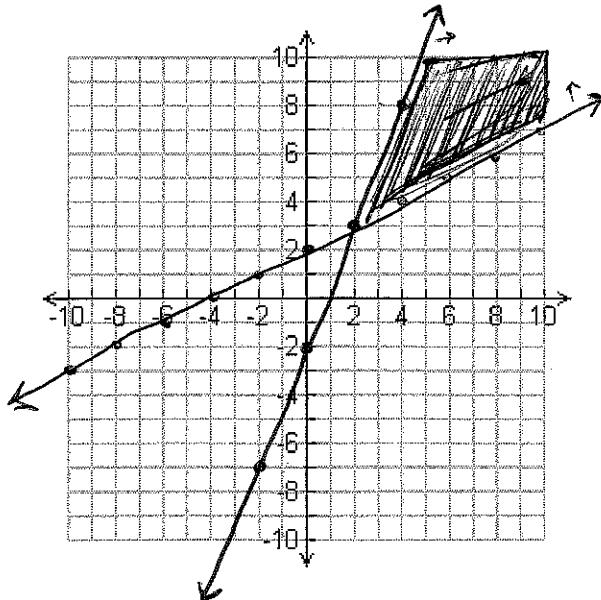
$$\begin{cases} 30 + 10x = y \\ 10 + 15x = y \end{cases}$$

$$\begin{aligned} 30 + 10x &= y \\ -10 + -15x &= -y \\ 20 + -5x &= 0 \\ x &= 4 \end{aligned}$$

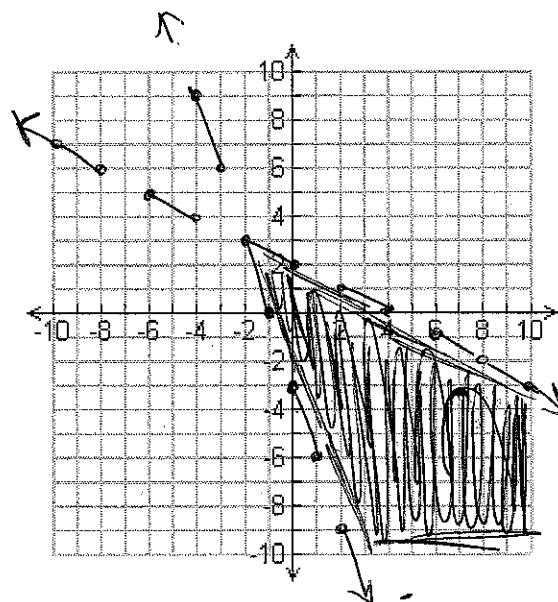
$$\begin{aligned} 30 + 10(4) &= y \\ 30 + 40 &= y \\ 70 &= y \end{aligned}$$

3.2: Solving Systems of inequalities by Graphing

5) $y \leq \frac{5}{2}x - 2$ $0 \leq -2 \times \textcircled{x}$
 $y \geq \frac{1}{2}x + 2$ $0 \geq 2 \times \textcircled{x}$



6) $3x + y > -3 \rightarrow y > -3x - 3$ $0 > -3 \quad \textcircled{v}$
 $x + 2y < 4 \rightarrow y < -\frac{1}{2}x + 2$ $0 < 2 \quad \textcircled{v}$



3.5: Operations with Matrices

Perform the indicated operations. If the matrix does not exist, write impossible.

8)
$$\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

10)
$$-3 \begin{bmatrix} 4a \\ 0 \\ -3 \end{bmatrix} + 4 \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -12a - 8 \\ 12 \\ 5 \end{bmatrix}$$

9)
$$3 \left(\begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix} \right)$$

$$\begin{bmatrix} -3 & 27 \\ 9 & 12 \end{bmatrix}$$

11)

$$\begin{bmatrix} -5 & 7 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 9 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

Not possible

3.6: Multiplying Matrices

Find each product, if possible.

11)
$$\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix} = \begin{bmatrix} 62 \end{bmatrix}$$

12)
$$\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} -12 & 17 \\ 82 & 26 \end{bmatrix}$$

13)
$$\begin{bmatrix} 2 & 11 \\ 0 & -3 \\ -6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & -5 \\ 12 & 0 & 9 \\ 4 & -6 & 7 \end{bmatrix} = \begin{bmatrix} \text{Not possible} \end{bmatrix}$$

14)
$$\begin{bmatrix} 2 & 0 \\ -3 & 5 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -19 \\ -5 \end{bmatrix}$$

3.8 Solving Systems of Equations using Inverse Matrices

Find the inverse of each matrix, if it exists.

15)
$$\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 \\ -1.5 & 3.5 \end{bmatrix}$$

16)
$$\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix} \quad \begin{bmatrix} 13 & 5 \\ -5 & -2 \end{bmatrix}$$

17)
$$\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix} \quad \begin{bmatrix} \text{No inverse} \end{bmatrix}$$

Use inverse matrices to solve the following system of equations.

18)
$$5x + 3y = 4$$

$$3x + 2y = 0$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} x = 8 \\ y = -12 \end{bmatrix}$$

19)
$$3a - b = 5$$

$$a + 2b = 4$$

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} a = 2 \\ b = 1 \end{bmatrix}$$

Section 4.1 - Graphing Quadratic Functions

For #1-4, find the y-intercept, the equation for the axis of symmetry, and the coordinates for the vertex.

$$1) f(x) = 2x^2 + 10x + 24$$

y-int: 24

AoS: $x = \frac{-10}{2 \cdot 2} = \frac{-5}{2} = -2.5$

Vertex: $(-2.5, 11.5)$

$$3) f(x) = -2x^2 + 6x - 2$$

y-int: -2

AoS: $x = \frac{-6}{2 \cdot -2} = \frac{6}{4} = 1.5$

Vertex: $(1.5, 2.5)$

Determine whether each function has a maximum or a minimum. Then, find the maximum/minimum value. \rightarrow vertex!

$$5) f(x) = -x^2 + 3x - 1 \quad x = \frac{-3}{2} = 1.5$$

Maximum
 $(1.5, 1.25)$

$$a < 0$$

$$a > 0$$

$$6) f(x) = -3x^2 - 4x + 5$$

$$x = \frac{4}{2 \cdot -3} = \frac{4}{-6}$$

Maximum
 $(-0.67, 9.0267)$

$$\begin{aligned} &-6(2)^2 + 24(2) - 2 \\ &-24 + 48 - 2 \\ &22 \end{aligned}$$

Section 4.2 - Solving Quadratic Equations by Graphing

Solve each equation by graphing. (hint: press 2ND, TRACE, to find the exact values for the zeros that are between two numbers!)

$$7) x^2 - x - 20 = 0$$

~~EE~~ $x = 5$
 $x = -4$

$$8) 4x^2 - 6x - 15 = 0$$

$x = -1.33$
 $x = 2.83$

Section 4.4 - Complex Numbers

Simplify.

$$11) \sqrt{-8}$$

$$\begin{aligned} &\sqrt{-1} \cdot \sqrt{8} \\ &i \cdot \sqrt{4 \cdot 2} \\ &2i\sqrt{2} \end{aligned}$$

$$14) (6 + 5i)(3 - 2i)$$

$$18 - 12i + 15i - 10i^2$$

$$\begin{aligned} &18 + 3i + 10 \\ &28 + 3i \end{aligned}$$

$$12) (2 - i) + (13 + 4i)$$

$$15 + 3i$$

$$13) (6 + 2i) - (4 - 3i)$$

$$2 + 5i$$

$$15) (-10 - 4i)(3 - 2i)$$

$$\begin{aligned} &-30 + 20i - 12i + 8i^2 \\ &-30 + 8i - 8 \\ &-38 + 8i \end{aligned}$$

Section 4.5 - Completing the Square

$$C = \left(\frac{b}{2}\right)^2$$

Find the value of c that makes each trinomial a perfect square. Then, write the trinomial as a perfect square.

16) $x^2 + 18x + c$

$$\begin{aligned} \left(\frac{18}{2}\right)^2 &= c \\ 81 &= c \\ X^2 + 18x + 81 & \end{aligned}$$

17) $x^2 - 4x + c$

$$\begin{aligned} \left(\frac{-4}{2}\right)^2 &= c \\ 4 &= c \\ X^2 - 4x + 4 & \end{aligned}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Section 4.6 - The Quadratic Formula and the Discriminant

For #22-24, Find the value of the discriminant, describe the number and type of roots, and find the solutions by using the quadratic formula. $\rightarrow b^2 - 4ac$

20) $x^2 - 2x + 9 = 0$

$$(-2)^2 - 4(1)(9)$$

$$\begin{aligned} -32 \\ 2 \text{ complex} \\ \text{Solutions} \\ X = \frac{2 \pm \sqrt{-32}}{2} \end{aligned}$$

21) $2x^2 + 19x - 33 = 0$

$$361 - 4(2)(-33)$$

$$\frac{-19 \pm 25}{4}$$

$$\begin{aligned} 625 \\ 2 \text{ real} \\ \text{Solutions} \\ X = 1.5 \\ X = -11 \end{aligned}$$

22) $x^2 - 10x + 25 = 0$

$$100 - 4(1)(25)$$

$$\begin{aligned} 0 \\ 1 \text{ real} \\ \text{Solution} \\ X = -5 \end{aligned}$$

Solve each equation by factoring, the quadratic formula, or completing the square.

9) $2x^2 - 2x - 24 = 0$

$$\begin{array}{c} \cancel{-48} \\ \cancel{-8} \quad 6 \\ \cancel{-2} \\ x \quad 3 \\ \hline 2x \quad 2x^2 \quad 6x \\ -8 \quad -8x \quad -24 \end{array}$$

$$(x+3)(2x-8)=0$$

$$X = -3 \quad X = 4$$

10) $2x^2 - 5x - 3 = 0$

$$5 \pm \sqrt{25 - 4(2)(-3)}$$

$$2 \cdot 2$$

$$\frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}$$

$$\begin{aligned} X &= 3 \\ X &= -1/2 \end{aligned}$$