

5-1 Study Guide and Intervention**Operations with Polynomials**

Multiply and Divide Monomials Negative exponents are a way of expressing the multiplicative inverse of a number.

Negative Exponents	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer n .
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When you **simplify an expression**, you rewrite it without powers of powers, parentheses, or negative exponents. Each base appears only once, and all fractions are in simplest form. The following properties are useful when simplifying expressions.

Product of Powers	$a^m \cdot a^n = a^{m+n}$ for any real number a and integers m and n .
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers m and n .
Properties of Powers	For a, b real numbers and m, n integers: $(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}, a \neq 0, b \neq 0$

Example

Simplify. Assume that no variable equals 0.

a. $(3m^4n^{-2})(-5mn)^2$

$$\begin{aligned}(3m^4n^{-2})(-5mn)^2 &= 3m^4n^{-2} \cdot 25m^2n^2 \\ &= 75m^4m^2n^{-2}n^2 \\ &= 75m^{4+2}n^{-2+2} \\ &= 75m^6\end{aligned}$$

b.
$$\begin{aligned}\frac{(-m^4)^3}{(2m^2)^{-2}} &= \frac{(-m^4)^3}{(2m^2)^{-2}} \\ &= \frac{(-m^{12})}{(2m^4)} \\ &= -m^{12} \cdot 4m^4 \\ &= -4m^{16}\end{aligned}$$

Exercises

Simplify. Assume that no variable equals 0.

1. $c^{12} \cdot c^{-4} \cdot c^6$

C
14

4. $\frac{x^{-2}y}{x^4y^{-1}}$

X
Y
6

7. $\frac{1}{2}(-5a^2b^3)^2(abc)^2$

12.5a⁶b⁸c²

10. $\frac{2^3c^4t^2}{2^2c^4t^2}$

2

2. $\frac{b^8}{b^2}$

b
6

5. $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$

b
a⁵

8. $m^7 \cdot m^8$

m
15

11. $4j(-j^{-2}k^2)(3j^3k^{-7})$

-12j²
K⁵

3. $(a^4)^5$

a
20

6. $\left(\frac{x^2y}{xy^3}\right)^2$

X²
Y⁴

9. $\frac{8m^3n^2}{4mn^3}$

2m²
n

12. $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$

1.5m²

5-1 Study Guide and Intervention

(continued)

Operations with Polynomials**Operations with Polynomials**

Polynomial	a monomial or a sum of monomials
Like Terms	terms that have the same variable(s) raised to the same power(s)

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1 Simplify $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$.

$$\begin{aligned} & 4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y) \\ &= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y \quad \text{Distribute the minus sign.} \\ &= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy) \quad \text{Group like terms.} \\ &= x^2y - xy^2 - 8xy \quad \text{Combine like terms.} \end{aligned}$$

You use the distributive property when you multiply polynomials. When multiplying binomials, the **FOIL** pattern is helpful.

FOIL Pattern	To multiply two binomials, add the products of F the <i>first</i> terms, O the <i>outer</i> terms, I the <i>inner</i> terms, and L the <i>last</i> terms.
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Example 2 Find $(6x - 5)(2x + 1)$.

$$\begin{aligned} (6x - 5)(2x + 1) &= 6x \cdot 2x + 6x \cdot 1 + (-5) \cdot 2x + (-5) \cdot 1 \\ &\quad \text{First terms} \quad \text{Outer terms} \quad \text{Inner terms} \quad \text{Last terms} \\ &= 12x^2 + 6x - 10x - 5 \quad \text{Multiply monomials.} \\ &= 12x^2 - 4x - 5 \quad \text{Add like terms.} \end{aligned}$$

Exercises**Simplify.**

1. $(6x^2 - 3x + 2) - (4x^2 + x - 3)$

$$2x^2 - 4x + 5$$

3. $(-4m^2 - 6m) - (6m + 4m^2)$

$$-8m^2 - 12m$$

5. $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$

$$-\frac{1}{8}x^2 - \frac{7}{8}xy + \frac{3}{4}y^2$$

2. $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$

$$3y^2 + 18xy - 8x^2$$

4. $27x^2 - 5y^2 + 12y^2 - 14x^2$

$$13x^2 + 7y^2$$

6. $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$

$$9p^3 - 2p^2 - 4p$$

Find each product.

7. $2x(3x^2 - 5)$

$$6x^3 - 10x$$

9. $(x^2 - 2)(x^2 - 5)$

$$x^4 - 7x^2 + 10$$

11. $(2n^2 - 3)(n^2 + 5n - 1)$

$$2n^4 + 10n^3 - 5n^2 - 15n + 3$$

8. $7a(6 - 2a - a^2)$

$$42a - 14a^2 - 7a^3$$

10. $(x + 1)(2x^2 - 3x + 1)$

$$2x^3 - x^2 - 2x + 1$$

12. $(x - 1)(x^2 - 3x + 4)$

$$x^3 - 4x^2 + 7x - 4$$

5-2 Study Guide and Intervention

Dividing Polynomials

Long Division To divide a polynomial by a monomial, use the skills learned in Lesson 5-1.

To divide a polynomial by a polynomial, use a long division pattern. Remember that only like terms can be added or subtracted.

Example 1

Simplify $\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr}$.

$$\begin{aligned} \frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr} &= \frac{12p^3t^2r}{3p^2tr} - \frac{21p^2qtr^2}{3p^2tr} - \frac{9p^3tr}{3p^2tr} \\ &= \frac{12}{3}p^{(3-2)}t^{(2-1)}r^{(1-1)} - \frac{21}{3}p^{(2-2)}qtr^{(1-1)}r^{(2-1)} - \frac{9}{3}p^{(3-2)}t^{(1-1)}r^{(1-1)} \\ &= 4pt - 7qr - 3p \end{aligned}$$

Example 2

Use long division to find $(x^3 - 8x^2 + 4x - 9) \div (x - 4)$.

$$\begin{array}{r} x^2 - 4x - 12 \\ x - 4 \overline{)x^3 - 8x^2 + 4x - 9} \\ (-)x^3 - 4x^2 \\ \hline -4x^2 + 4x \\ (-) -4x^2 + 16x \\ \hline -12x - 9 \\ (-) -12x + 48 \\ \hline -57 \end{array}$$

The quotient is $x^2 - 4x - 12$, and the remainder is -57 .

Therefore $\frac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \frac{57}{x - 4}$.

Exercises

Simplify.

1. $\frac{18a^3 + 30a^2}{3a}$

$$(6a^2 + 10a)$$

4. $(2x^2 - 5x - 3) \div (x - 3)$

$$(2x+1) R:0$$

6. $(p^3 - 6) \div (p - 1)$

8. $(x^5 - 1) \div (x - 1)$

2. $\frac{24mn^6 - 40m^2n^3}{4m^2n^3}$

$$\left(\frac{6n^3}{m} - 10\right)$$

3. $\frac{60a^2b^3 - 48b^4 + 84a^5b^2}{12ab^2}$

$$\left(5ab - \frac{4b^2}{a} + 7a^4\right)$$

5. $(m^2 - 3m - 7) \div (m + 2)$

$$m-5 R:3$$

7. $(t^3 - 6t^2 + 1) \div (t + 2)$

$$(2x^2 - x + 2) R:0$$

9. $(2x^3 - 5x^2 + 4x - 4) \div (x - 2)$

6-1 Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Operations with Functions

1, 2

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.

$$(f + g)(x) = f(x) + g(x)$$

Addition of functions

$$= (x^2 + 3x - 4) + (3x - 2)$$

$$= x^2 + 6x - 6$$

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

Simplify.

$$(f - g)(x) = f(x) - g(x)$$

Subtraction of functions

$$= (x^2 + 3x - 4) - (3x - 2)$$

$$= x^2 - 2$$

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

Simplify.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Multiplication of functions

$$= (x^2 + 3x - 4)(3x - 2)$$

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

$$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$$

Distributive Property

$$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$$

Distributive Property

$$= 3x^3 + 7x^2 - 18x + 8$$

Simplify.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Division of functions

$$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$$

$$f(x) = x^2 + 3x - 4 \text{ and } g(x) = 3x - 2$$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 8x - 3; g(x) = 4x + 5$

$$(f+g)(x) = 12x+2 \quad (f/g)(x) = \frac{8x-3}{4x+5}$$

$$(f-g) = 4x - 8$$

$$(f \cdot g) = 32x^2 + 28x - 15$$

3. $f(x) = 3x^2 - x + 5; g(x) = 2x - 3$

$$(f+g) = 3x^2 + x + 2$$

$$(f-g) = 3x^2 - 3x + 8$$

$$(f \cdot g) = 6x^3 - 11x^2 + 13x - 15$$

$$(f/g) = 3x^2 - x + 5 / 2x - 3$$

5. $f(x) = x^2 - 1; g(x) = \frac{1}{x+1}$

$$(f+g) = x^2 - 1 + \frac{1}{x+1} \quad (f \cdot g) = \frac{x^2 - 1}{x+1}$$

$$(f-g) = x^2 - 1 - \frac{1}{x+1} \quad (f/g) = (x^2 - 1) \div \left(\frac{1}{x+1}\right)$$

2. $f(x) = x^2 + x - 6; g(x) = x - 2$

$$(f+g) \rightarrow x^2 + 2x - 8 \quad (f/g) \rightarrow \frac{x^2 + x - 6}{x-2}$$

$$(f-g) \rightarrow x^2 - 4$$

$$(f \cdot g) \rightarrow x^3 - x^2 - 8x + 12$$

4. $f(x) = 2x - 1; g(x) = 3x^2 + 11x - 4$

$$(f+g) = 3x^2 + 13x - 5$$

$$(f-g) = -3x^2 - 9x + 3$$

$$(f \cdot g) = 6x^3 + 19x^2 - 19x + 4$$

$$(f/g) = \frac{2x - 1}{3x^2 + 11x - 4}$$

6-1 Study Guide and Intervention

(continued)

Operations on Functions

3, 5, 4, 6 Tues

Composition of Functions Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

Example 1 For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.

$$f[g(1)] = f(3) = 3 \quad f[g(2)] = f(2) = 4 \quad f[g(3)] = f(4) = 1 \quad f[g(4)] = f(1) = 2,$$

$$\text{So } f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$$

$$g[f(1)] = g(2) = 2 \quad g[f(2)] = g(4) = 1 \quad g[f(3)] = g(3) = 4 \quad g[f(4)] = g(1) = 3,$$

$$\text{So } g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

Example 2 Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.

$$\begin{aligned} [g \circ h](x) &= g[h(x)] & [h \circ g](x) &= h[g(x)] \\ &= g(x^2 - 1) & &= h(3x - 4) \\ &= 3(x^2 - 1) - 4 & &= (3x - 4)^2 - 1 \\ &= 3x^2 - 7 & &= 9x^2 - 24x + 16 - 1 \\ & & &= 9x^2 - 24x + 15 \end{aligned}$$

Exercises

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist.

1. $f = \{(-1, 2), (5, 6), (0, 9)\},$
 $g = \{(6, 0), (2, -1), (9, 5)\}$

2. $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\},$
 $g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist.

3. $f(x) = 2x + 7; g(x) = -5x - 1$

$$f(g(x)) = -10x + 5$$

$$g(f(x)) = -10x - 36$$

5. $f(x) = x^2 + 2x; g(x) = x - 9$

$$f(g(x)) = x^2 - 16x + 63$$

$$g(f(x)) = x^2 + 2x - 9$$

4. $f(x) = x^2 - 1; g(x) = -4x^2$

$$f(g(x)) = 16x^4 - 1$$

$$g(f(x)) = -4x^4 + 8x^2 - 4$$

6. $f(x) = 5x + 4; g(x) = 3 - x$

$$f(g(x)) = 19 - 5x$$

$$g(f(x)) = -1 - 5x$$

6-2 Study Guide and Intervention

Inverse Functions and Relations

Tutor

Find Inverses

Inverse Relations Two relations are inverse relations if and only if whenever one relation contains the element (a, b) , the other relation contains the element (b, a) .
Property of Inverse Functions Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

$$\text{Inverse of } y = \frac{2}{5}x - \frac{1}{5}$$

$$5x = 2y - 1$$

Multiply each side by 5.

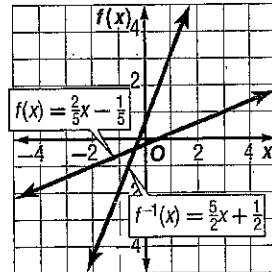
$$5x + 1 = 2y$$

Add 1 to each side.

$$\frac{1}{2}(5x + 1) = y$$

Divide each side by 2.

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

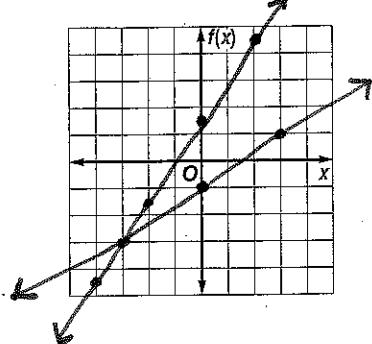


Exercises

Find the inverse of each function. Then graph the function and its inverse.

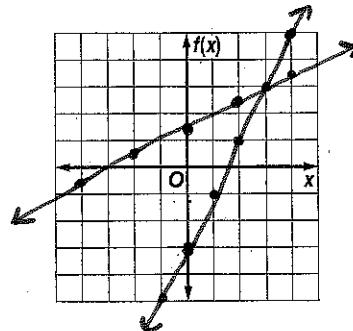
$$1. f(x) = \frac{2}{3}x - 1$$

$$f^{-1}(x) = \frac{3}{2}x + \frac{3}{2}$$



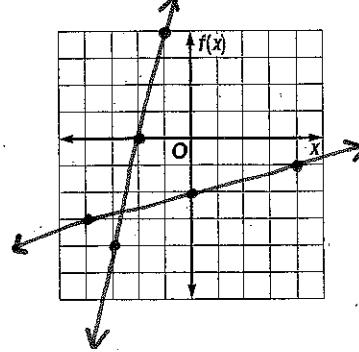
$$2. f(x) = 2x - 3$$

$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



$$3. f(x) = \frac{1}{4}x - 2$$

$$f^{-1}(x) = 4x + 8$$



6-2 Study Guide and Intervention

(continued)

Inverse Functions and Relations

1-6

TUES

Verifying Inverses**Inverse Functions**Two functions $f(x)$ and $g(x)$ are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.

$$[f \circ g](x) = f[g(x)]$$

$$= f\left[\frac{1}{2}(x + 7)\right]$$

$$= 2\left[\frac{1}{2}(x + 7)\right] - 7$$

$$= x + 7 - 7$$

$$= x$$

$$[g \circ f](x) = g[f(x)]$$

$$= g(2x - 7)$$

$$= \frac{1}{2}(2x - 7 + 7)$$

$$= x$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.

$$[f \circ g](x) = f[g(x)]$$

$$= f\left(\frac{1}{4}x - 3\right)$$

$$= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3}$$

$$= x - 12 + \frac{1}{3}$$

$$= x - 11\frac{2}{3}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

$$\begin{aligned} y &= 3x - 1 \\ x &= 3y - 1 \\ x + 1 &= 3y \\ \frac{1}{3}x + \frac{1}{3} &= y \end{aligned}$$

Exercises

Determine whether each pair of functions are inverse functions. Write yes or no.

1. $f(x) = 3x - 1$
 $g(x) = \frac{1}{3}x + \frac{1}{3}$ Yes

2. $f(x) = \frac{1}{4}x + 5$
 $g(x) = 4x - 20$ Yes

3. $f(x) = \frac{1}{2}x - 10$
 $g(x) = 2x + \frac{1}{10}$ No

4. $f(x) = 2x + 5$
 $g(x) = 5x + 2$ No

5. $f(x) = 8x - 12$
 $g(x) = \frac{1}{8}x + 12$ No

6. $f(x) = -2x + 3$
 $g(x) = -\frac{1}{2}x + \frac{3}{2}$ Yes

7. $f(x) = 4x - \frac{1}{2}$
 $g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$
 $g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$
 $g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$
 $g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$
 $g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$
 $g(x) = \frac{2}{3}x - 6$

6-3 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a **square root function**. The domain of a square root function is those values for which the radicand is greater than or equal to 0.

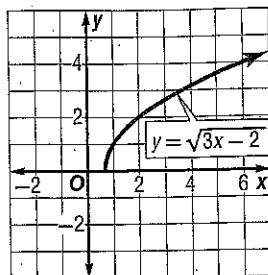
Example Graph $y = \sqrt{3x - 2}$. State its domain and range.

Since the radicand cannot be negative, the domain of the function is $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x -intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

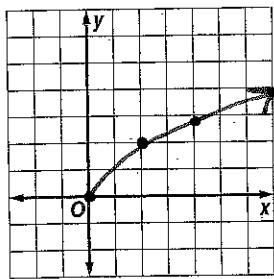
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



Exercises

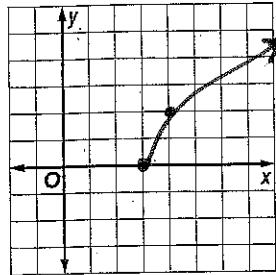
Graph each function. State the domain and range.

1. $y = \sqrt{2x}$



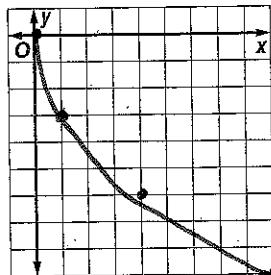
D: $x \geq 0$
R: $y \geq 0$

4. $y = 2\sqrt{x - 3}$



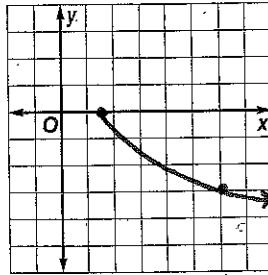
D: $x \geq 3$
R: $y \geq 0$

2. $y = -3\sqrt{x}$



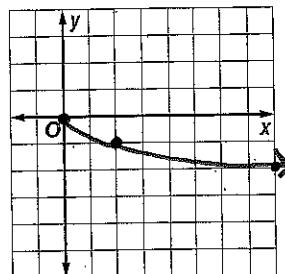
D: $x \geq 0$
R: $y \leq 0$

5. $y = -\sqrt{2x - 3}$



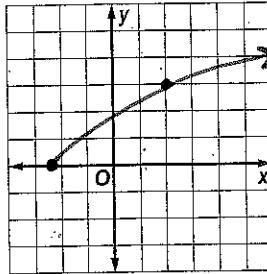
D: $x \geq \frac{3}{2}$
R: $y \leq 0$

3. $y = -\sqrt{\frac{x}{2}}$



D: $x \geq 0$
R: $y \leq 0$

6. $y = \sqrt{2x + 5}$



D: $x \geq -2.5$
R: $y \geq 0$

6-4 Study Guide and Intervention***nth Roots*****Simplify Radicals**

Wednesday

Square Root	For any real numbers a and b , if $a^2 = b$, then a is a square root of b .
<i>n</i>th Root	For any real numbers a and b , and any positive integer n , if $a^n = b$, then a is an n th root of b .
Real nth Roots of b, $\sqrt[n]{b}, -\sqrt[n]{b}$	<ol style="list-style-type: none"> If n is even and $b > 0$, then b has one positive real root and one real negative root. If n is odd and $b > 0$, then b has one positive real root. If n is even and $b < 0$, then b has no real roots. If n is odd and $b < 0$, then b has one negative real root.

Example 1 Simplify $\sqrt{49z^8}$.

$$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 Simplify $-\sqrt[3]{(2a-1)^6}$

$$-\sqrt[3]{(2a-1)^6} = -\sqrt[3]{[(2a-1)^2]^3} = -(2a-1)^2$$

Exercises**Simplify.**

1. $\sqrt{81}$
9

2. $\sqrt[3]{-343}$
-7

3. $\sqrt[3]{144p^6}$
 $12p^3$

4. $\pm\sqrt{4a^{10}}$
 $\pm 2a^5$

5. $\sqrt[5]{243p^{10}}$
 $3p^2$

6. $-\sqrt[3]{m^6n^9}$
 $-m^2n^3$

7. $\sqrt[3]{-b^{12}}$
 $-b^4$

8. $\sqrt{16a^{10}b^8}$
 $4a^5b^4$

9. $\sqrt{121x^6}$
 $11x^3$

10. $\sqrt{(4k)^4}$
 $16k^2$

11. $\pm\sqrt{169r^4}$
 $\pm 13r^2$

12. $-\sqrt[3]{-27p^6}$
 $3p^2$

13. $-\sqrt{625y^2z^4}$
 $-25yz^2$

14. $\sqrt{36q^{34}}$
 $6q^{17}$

15. $\sqrt{100x^2y^4z^6}$
 $10xy^2z^3$

16. $\sqrt[3]{-0.027}$
 -0.3

17. $-\sqrt{-0.36}$
0

18. $\sqrt{0.64p^{10}}$
 $0.8p^5$

19. $\sqrt[4]{(2x)^8}$
 $4x^2$

20. $\sqrt{(11y^2)^4}$
 $121y^4$

21. $\sqrt[3]{(5a^2b)^6}$
 $25a^4b^2$

22. $\sqrt{(3x-1)^2}$
 $3x-1$

23. $\sqrt[3]{(m-5)^6}$
 $(m-5)^2$

24. $\sqrt{36x^2 - 12x + 1}$
SKIP

6-4**Study Guide and Intervention**

(continued)

nth Roots**Approximate Radicals with a Calculator****Irrational Number**

a number that cannot be expressed as a terminating or a repeating decimal

Radicals such as $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications. These approximations can be easily found with a calculator.

ExampleUse a calculator to approximate $\sqrt[5]{18.2}$ to three decimal places.

$$\sqrt[5]{18.2} \approx 1.787$$

Exercises

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{62}$

7.874

2. $\sqrt{1050}$

32.404

3. $\sqrt[3]{0.054}$

0.378

4. $-\sqrt[4]{5.45}$

-1.528

5. $\sqrt{5280}$

72.664

6. $\sqrt{18,600}$

136.382

7. $\sqrt{0.095}$

0.308

8. $\sqrt[3]{-15}$

-2.466

9. $\sqrt[5]{100}$

2.512

10. $\sqrt[6]{856}$

3.081

11. $\sqrt{3200}$

56.569

12. $\sqrt{0.05}$

0.224

13. $\sqrt[7]{12,500}$

111.803

14. $\sqrt{0.60}$

0.775

15. $-\sqrt[4]{500}$

-4.729

16. $\sqrt[3]{0.15}$

0.531

17. $\sqrt[6]{4200}$

4.017

18. $\sqrt{75}$

8.660

19. **LAW ENFORCEMENT** The formula $r = 2\sqrt{5L}$ is used by police to estimate the speed r in miles per hour of a car if the length L of the car's skid mark is measured in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long.

77.5 mi
hr

20. **SPACE TRAVEL** The distance to the horizon d miles from a satellite orbiting h miles above Earth can be approximated by $d = \sqrt{8000h + h^2}$. What is the distance to the horizon if a satellite is orbiting 150 miles above Earth? ≈ 1100 mi

6-7 Study Guide and Intervention**Solving Radical Equations and Inequalities**

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

- Step 1** Isolate the radical on one side of the equation.
- Step 2** To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
- Step 3** Solve the resulting equation.
- Step 4** Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1 Solve $2\sqrt{4x+8} - 4 = 8$.

$$\begin{aligned} 2\sqrt{4x+8} - 4 &= 8 && \text{Original equation} \\ 2\sqrt{4x+8} &= 12 && \text{Add 4 to each side.} \\ \sqrt{4x+8} &= 6 && \text{Isolate the radical.} \\ 4x+8 &= 36 && \text{Square each side.} \\ 4x &= 28 && \text{Subtract 8 from each side.} \\ x &= 7 && \text{Divide each side by 4.} \end{aligned}$$

Check

$$\begin{aligned} 2\sqrt{4(7)+8} - 4 &\stackrel{?}{=} 8 \\ 2\sqrt{36} - 4 &\stackrel{?}{=} 8 \\ 2(6) - 4 &\stackrel{?}{=} 8 \\ 8 &= 8 \end{aligned}$$

The solution $x = 7$ checks.

Example 2 Solve $\sqrt{3x+1} = \sqrt{5x} - 1$.

$$\begin{aligned} \sqrt{3x+1} &= \sqrt{5x} - 1 && \text{Original equation} \\ 3x+1 &= 5x - 2\sqrt{5x} + 1 && \text{Square each side.} \\ 2\sqrt{5x} &= 2x && \text{Simplify.} \\ \sqrt{5x} &= x && \text{Isolate the radical.} \\ 5x &= x^2 && \text{Square each side.} \\ x^2 - 5x &= 0 && \text{Subtract } 5x \text{ from each side.} \\ x(x-5) &= 0 && \text{Factor.} \\ x = 0 \text{ or } x &= 5 && \end{aligned}$$

Check

$$\begin{aligned} \sqrt{3(0)+1} &= 1, \text{ but } \sqrt{5(0)} - 1 = -1, \text{ so } 0 \text{ is} \\ &\text{not a solution.} \\ \sqrt{3(5)+1} &= 4, \text{ and } \sqrt{5(5)} - 1 = 4, \text{ so the} \\ &\text{solution is } x = 5. \end{aligned}$$

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$

$x = 0.577$

2. $2\sqrt{3x+4} + 1 = 15$

15

3. $8 + \sqrt{x+1} = 2$

$\cancel{0}$

4. $\sqrt{5-x} - 4 = 6$

-95

5. $12 + \sqrt{2x-1} = 4$

$\cancel{0}$

6. $\sqrt{12-x} = 0$

12

7. $\sqrt{21} - \sqrt{5x-4} = 0$

7

8. $10 - \sqrt{2x} = 5$

12.5

9. $\sqrt{4+7x} = \sqrt{7x-9}$

$\cancel{0}$

10. $4\sqrt[4]{2x+11} - 2 = 10$

8

11. $2\sqrt{x-11} = \sqrt{x+4}$

16

12. $(9x-11)^{\frac{1}{2}} = x+1$

$3, 4$

6-7 Study Guide and Intervention

(continued)

Solving Radical Equations and Inequalities

Solve Radical Inequalities A radical inequality is an inequality that has a variable in a radicand. Use the following steps to solve radical inequalities.

1) V

- Step 1** If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.
Step 2 Solve the inequality algebraically.
Step 3 Test values to check your solution.

Example

$$\text{Solve } 5 - \sqrt{20x + 4} \geq -3.$$

Since the radicand of a square root must be greater than or equal to zero, first solve

$$20x + 4 \geq 0.$$

$$20x + 4 \geq 0$$

$$20x \geq -4$$

$$x \geq -\frac{1}{5}$$

Now solve $5 - \sqrt{20x + 4} \geq -3$.

$$5 - \sqrt{20x + 4} \geq -3 \quad \text{Original inequality}$$

$$\sqrt{20x + 4} \leq 8 \quad \text{Isolate the radical.}$$

$$20x + 4 \leq 64 \quad \text{Eliminate the radical by squaring each side.}$$

$$20x \leq 60 \quad \text{Subtract 4 from each side.}$$

$$x \leq 3 \quad \text{Divide each side by 20.}$$

Eva

1) L

4) li

Exr

7) lk

Cor

9) 4

Sol

11)

13)

It appears that $-\frac{1}{5} \leq x \leq 3$ is the solution. Test some values.

$x = -1$	$x = 0$	$x = 4$
$\sqrt{20(-1) + 4}$ is not a real number, so the inequality is not satisfied.	$5 - \sqrt{20(0) + 4} = 3$, so the inequality is satisfied.	$5 - \sqrt{20(4) + 4} \approx -4.2$, so the inequality is not satisfied.

Therefore the solution $-\frac{1}{5} \leq x \leq 3$ checks.

Exercises

Solve each inequality.

1. $\sqrt{c - 2} + 4 \geq 7$

$$C \geq 11$$

2. $3\sqrt{2x - 1} + 6 < 15$

$$\frac{1}{2} \leq x < 5$$

3. $\sqrt{10x + 9} - 2 > 5$

$$x > 4$$

4. $8 - \sqrt{3x + 4} \geq 3$

$$-4 \frac{1}{3} \leq x \leq 7$$

5. $\sqrt{2x + 8} - 4 > 2$

$$x > 14$$

6. $9 - \sqrt{6x + 3} \geq 6$

$$-\frac{1}{2} \leq x \leq 1$$

7. $2\sqrt{5x - 6} - 1 < 5$

$$6 \frac{1}{5} \leq x < 3$$

8. $\sqrt{2x + 12} + 4 \geq 12$

$$x \geq 26$$

9. $\sqrt{2d + 1} + \sqrt{d} \leq 5$